Graphing Rational Functions: A General Method

I. Factor everything that can be factored.

II. Find the domain by investigating the denominator (any roots of the denominator are NOT in the domain).

III. Reduce by canceling out any factors that appear in the numerator and denominator. Any factors that cancel out like this may lead to "holes" in the final graph.

IV. Find vertical asymptotes: any roots of the denominator in reduced form will be vertical asymptotes. Evaluate the function at x-values VERY close to the asymptote to determine behavior near asymptote.

V. Find intercepts. Evaluate the function's value at 0 to find the y-intercept; set the numerator equal to 0 and solve for x to find the x-intercept(s).

VI. Find horizontal/oblique asymptote:

   a. If the numerator's degree is *lower* than the denominator's degree, there is a *horizontal* asymptote at y=0.

   b. If the numerator's degree is *equal* to the denominator's degree, ignore everything except the highest degree terms and reduce; there is a *horizontal* asymptote at y=the result.

   c. If the numerator's degree is *one more* than the denominator's degree, there is an *oblique* (diagonal) asymptote. Use polynomial division to divide the fraction; drop any remainder, and what's left is the equation of the asymptote.

   d. If the numerator's degree exceeds the denominator's degree by *more than one*, there is no linear asymptote. However, dividing the fraction as in (c) will still yield a function that the rational function will "approach" when x is very large; it just won't be linear, so it doesn't count as an asymptote.

VII. Determine if the horizontal/oblique is ever intersected by the function by setting the function equal to the asymptote and solving for x. If there are no solutions, then the graph never crosses the asymptote. If there are solutions, then the graph crosses the asymptote at that x-value.

VIII. Determine end behavior by picking a few x-values *outside* of the outermost known points and calculating their y-values.

IX. Connect all known points with a smooth curve, being careful to respect both end behavior and behavior near asymptotes. Leave a hole at any x-value not in the domain, unless there's already an asymptote there.

X. Admire the beauty and power of the connectedness of all things in mathematicks.