EXPONENTIAL EQUATIONS

1) The function $P(t)=k\cdot e^{rt}$ can be used to describe opulation growth, radioactive decay, heating/cooling, fluid draining from a tank, and a variety of other scenarios.
   a) What determines whether this is growth or decay?

   b) What does k represent? (Hint: what value of t makes everything else vanish?)

2) Consider a tank that initially holds 100 liters of water.
   After 5 minutes of draining, there are only 75 liters left.
   a) Find values of k and r and write an equation for the amount of water at time t.

   b) How much water is left 8 minutes after the draining started?

   c) When will there be exactly 50 liters? ...25 liters? ...12.5 liters?

   d) According to this model, will the tank ever be empty? Is this realistic?

3) Suppose a study found that in a certain forest there lived 500 squirrels in the year 1920 and 1100 squirrels in the year 1925. Assuming exponential growth...
   a) Write an equation to model the population, treating the year 1900 as $t = 0$.

   b) What *was* the population in the year 1900?

   c) How long does it take the population to double?

   d) How many squirrels should we expect in 2020? Is this realistic? Why or why not?

   e) What *was* the population in 1800? Is this realistic? Why or why not?

4) Consider a population of squids that is growing, starting with 100 squids and doubling every 5 years, and a population of fish that is going extinct, starting with 3000 fish and halving every 7 years.
   a) Write an equation for each population, $S(t)$ and $F(t)$.

   b) When will there be the same number of fish and squids?

5) Entropy increases when heat is added to a system, and this change is usually calculated as $\Delta S=C\cdot\ln\left(\frac{T_f}{T_i}\right)$ where C is the object's heat capacity and T must be measured in kelvins.
   a) Suppose an object has $C = 2000$ J/K, and is currently at a temperature of 100 K. How high must the temperature rise to increase entropy by $\Delta S = 1000$ J/K ?

   b) What if the same object started at an initial temperature of 500 K instead?

QUADRATICS

6) A cannonball soaring through the sky has (x, y) coordinates described by

$$x(t)=x_0+v_{xi}\cdot t \qquad\qquad y(t)=y_0+v_{yi}\cdot t+\frac{g}{2}\cdot t^2$$

Suppose it is launched from the top of a 5m tall tower (on Earth, where $g = -9.8\ \text{m/s}^2$) with an initial velocity of 20 m/s in the x-direction and 15 m/s in the y-direction. When and where will it land? When and where will it reach its maximum height?

7) Ignoring time for the moment, a cannonball should travel along a parabola-shaped path, which can be described by the equation $y=ax^2+bx+c$
Suppose we want a trajectory that passes through the points (1,5), (3,17), and (10,0). Find values of the parameters a, b, and c that make this happen.
(Hint: Substitute each set of xy-values into the equation. What do you now have? Solve!)

8) Find a parabola that passes through the points (2,3), (5,18), and (6,23).

9) Find a parabola that passes through the points (4,5), (6,10), and (4,12).

SYSTEMS OF EQUATIONS

10) Chemical equations like __C_3H_8 + __O_2 —> __CO_2 + __H_2O (combustion of propane) can be solved by guess-and-check, but they can also be treated as systems of equations! What *don't* you know about this reaction? Assign a variable to each of those unknowns. What has to match up on both sides of the reaction? Set up an equation for each match. Now solve that system of equations.

11) These equations describe the electrical current in different parts of a circuit. Find values of I_1, I_2, and I_3 that make all of them true:

$$I_1+I_2=I_3 \qquad\qquad 12-6I_1-4I_3=0 \qquad\qquad 12-3I_2-2I_2-4I_3=0$$

FRACTIONS

12) The "thin lens equation" $\frac{1}{f}=\frac{1}{o}+\frac{1}{i}$ describes the relationship between the *f*ocal length of a lens, the location of the *o*bject, and the location of the *i*mage the lens creates. For each set of values for f and o, find the value of i.
(Hint: be lazy! You only need to solve the equation once!)

a) $f = 3, o = 10$ b) $f = 3, o = 2$ c) $f = 3, o = 3$

d) $f = -3, o = 3$ e) $f = -5, o = 50$ f) $f = 5, o = \infty$