INTEGRALS WORKSHEET ANSWER KEY

1) a) The (definite) integral of a function describes the area under its graph.

b) The (definite) integral of the rate of change calculates how much change has happened so far. The (indefinite) integral simply "undoes" the derivative.

c) One example: speed is the rate of change of position, so the (definite) integral of speed calculates how much the position has changed so far.

2) Δx is a small bit of x, so the $\sum$ means adding up all the tiny bits of f–times–Δx.
The limit turns an approximation into an exact value, using smaller and smaller Δx.
This also finds area under the graph by splitting it into rectangles (height x width).

3) $\frac{d}{dx}\left(\int f(x)\,dx\right)=f(x)$

4) $\int\left(\frac{d}{dx}f(x)\right)dx=f(x)+c$

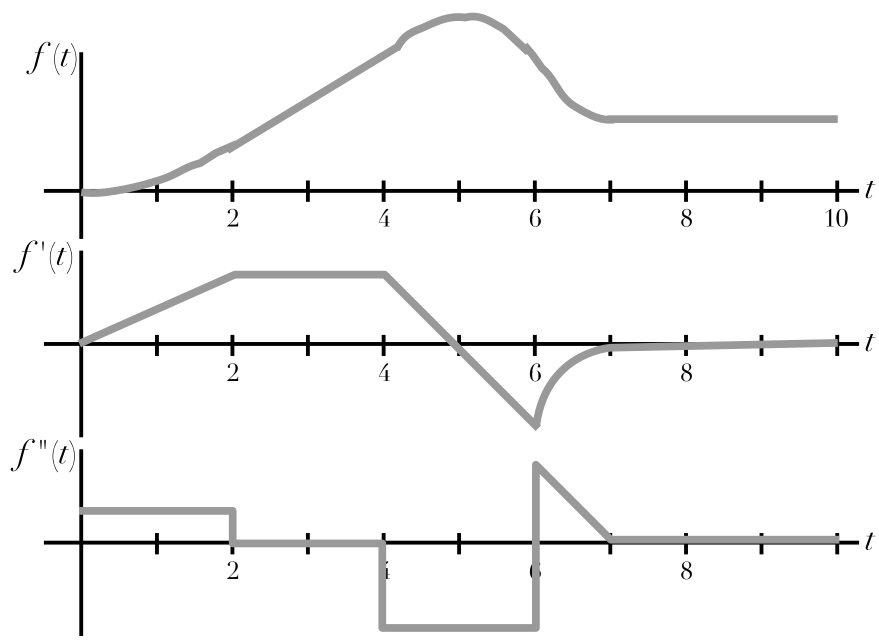
5) A *definite* integral calculates the area under the curve within a certain interval, or the amount of some quantity that has accumulated during some time, and is a *number*.

An *indefinite* integral results in a new *function* whose derivative is the original function inside the integral. This should include a "+ c" because we want all possible answers; we know the derivative of the answer, but we know nothing about its "starting point."

Another useful way to think of this: an *indefinite* integral asks "What's a function whose rate of change is this?" (so the answer is a function), whereas a *definite* integral further asks "…and how much has it changed *so far*?"

6) The definite integral is positive when (most of) the area is above the axis.
The definite integral is negative when (most of) the area is below the axis.
The definite integral is zero when the areas above and below the axis balance out.

7)



For 8–14 you can just count the "grid squares" (and fractional triangles).
Area *under* the curve counts as negative, and going *right-to-left* introduces another negative.

8) $\int_0^2 f(t)\,dt = 1.5$

9) $\int_1^3 f(t)\,dt = 0$

10) $\int_{10}^7 f(t)\,dt = -4.5$

11) $\int_7^2 f(t)\,dt = 5$

12) $\int_0^{10} f(t)\,dt = 1$

13) $\int_0^{10} \left(f(t)+5\right)dt = 51$

14) $\int_0^{10} \left(-3\cdot f(t)\right)dt = -3$

15) $\int 0\,dt = c$

16) $\int -5\,dt = -5t + c$

17) $\int_1^3 \left(9t^2+4t-6\right)dt = 82$

18) $\int_0^\infty e^{-3t} = \frac{1}{3}$

19) $\int \left(\frac{1}{t^2}+\frac{1}{t}\right)dt = \ln(t) - \frac{1}{t} + c$

20) $\int_0^2 \sqrt{4t+1}\,dt = 4\frac{1}{3}$

21) $\int \frac{5}{3t-4}\,dt = \frac{5}{3}\cdot\ln|3t-4| + c$

22) $\int t\cdot e^{5t^2}\,dt = \frac{1}{10}\cdot e^{5t^2} + c$

23) $y = C\cdot e^{5t} - \frac{3}{5}$