INTEGRALS: WHAT THEY MEAN

1) Explain in your own words what "the integral of a function" actually means...
   a) ...for a graph of the function.
   b) ...regarding the rate of change of the function.
   c) ...for a physical situation represented by a function (pick your own example).

2) The mathematical definition of the integral is

$$\int_a^b f(x)dx = \lim_{\Delta x \to 0} \sum f(x_n) \cdot \Delta x$$

   What does each piece of this definition mean?
   How does it compare to your descriptions in questions 1a, 1b, and 1c?

3) If you take the integral of a function, and then take the derivative, what do you get?

$$\frac{d}{dx}\left(\int f(x)\,dx\right) =$$

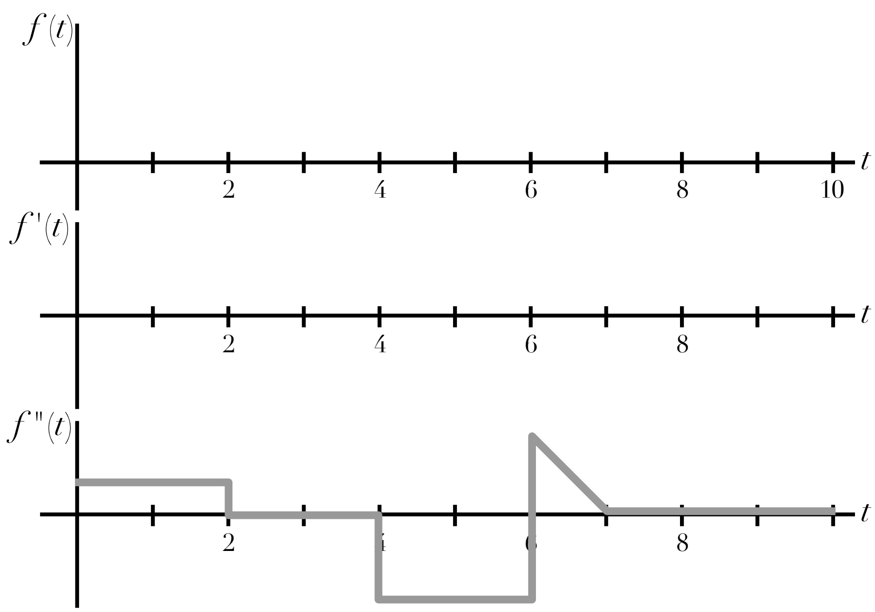
4) If you take the derivative of a function, and then take the integral, what do you get?

$$\int \left(\frac{d}{dx} f(x)\right) dx =$$

5) Describe the difference between a *definite* integral and an *indefinite* integral.
   One of them always includes a "+ *c*"; what does this mean, and why is it necessary?

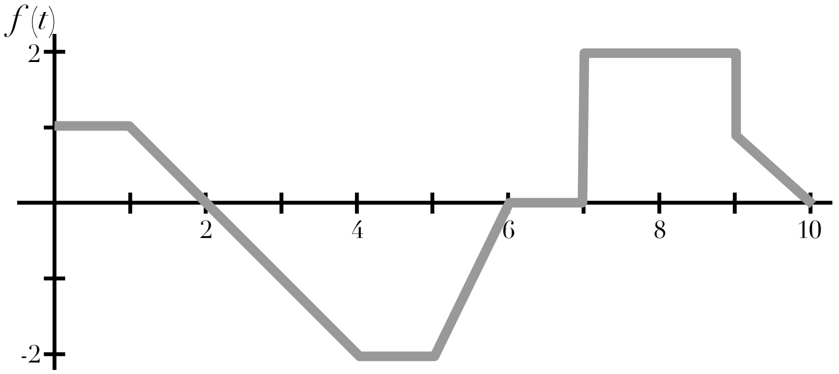
6) What does it mean when the value of a definite integral is zero? ...positive? ...negative?

7) Given this graph of the *second* derivative of some mystery function, sketch possible graphs of the function's first derivative and the function itself.



INTEGRALS: CALCULATIONS

Given this graph of $f(t)$, evaluate each definite integral. (Hint: think geometrically!)



8) $\int_0^2 f(t)\,dt =$

9) $\int_1^3 f(t)\,dt =$

10) $\int_{10}^7 f(t)\,dt =$

11) $\int_7^2 f(t)\,dt =$

12) $\int_0^{10} f(t)\,dt =$

13) $\int_0^{10} (f(t)+5)\,dt =$

14) $\int_0^{10} (-3 \cdot f(t))\,dt$

Evaluate the following integrals:

15) $\int 0\,dt =$

16) $\int -5\,dt =$

17) $\int_1^3 (9t^2+4t-6)\,dt =$

18) $\int_0^\infty e^{-3t} =$

19) $\int \left(\frac{1}{t^2}+\frac{1}{t}\right)dt =$

20) $\int_0^2 \sqrt{4t+1}\,dt =$

21) $\int \frac{5}{3t-4}\,dt =$

22) $\int t \cdot e^{5t^2}\,dt =$

23) Suppose that all you know about a function is that its derivative is 3 more than 5 times its own value: $\frac{dy}{dt} = 5y+3$. Use integrals to find the function y. (Hint: multiply by dt.)