

## DERIVATIVES: WHAT THEY MEAN

- 1) Explain in your own words what “the derivative of a function” actually means...
- ...for a graph of the function.
  - ...for the input and output of the function.
  - ...for a physical situation represented by a function (pick your own example).

- 2) The mathematical definition of the derivative is

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad \text{which could also be written as} \quad \frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x}$$

What does each piece of this definition mean?

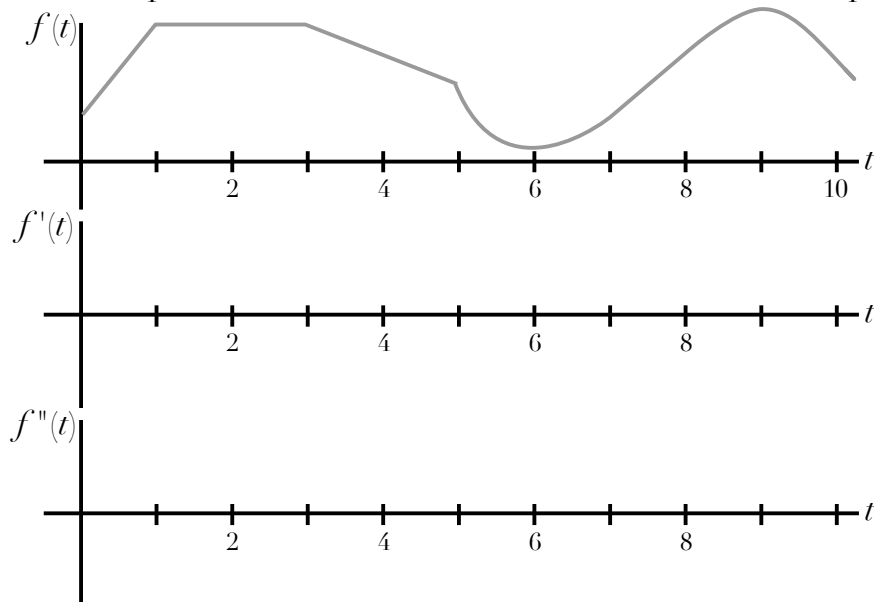
How does it compare to your descriptions in questions 1a, 1b, and 1c?

- 3) What is the difference between a *secant* line and a *tangent* line on the graph of a function?  
How does this relate to your answers to question 1a?

- 4) What does the *second* derivative  $\left(\frac{d^2 f}{dx^2}\right)$  tell you about a graph?

- 5) If the derivative is zero for some  $x$ -value, what’s happening at that point?  
If the second derivative is zero for some  $x$ -value, what's happening at that point?

- 6) Given this graph of a function, sketch graphs of its first derivative and second derivative.  
Exact value isn’t important; focus on where the derivatives are zero, positive, or negative.



- 7) If the original function represents *position* of an object that is moving as time passes, what do the first and second derivative represent?

- 8) Describe in detail what the object is *doing* at times  $t = 2$ ,  $t = 4$ ,  $t = 6$ , and  $t = 9$ .

## DERIVATIVES: CALCULATIONS

Find the first derivative of each of the following functions:

9)  $f(t)=5$

10)  $f(t)=3t^4$

11)  $f(t)=5t^2-3t+4$

12)  $f(t)=\sqrt{t^3}$

13)  $f(t)=\frac{3}{t^{12}}$

14)  $f(t)=-4e^t$

15)  $f(t)=6t^5 \cdot e^t$

16)  $f(t)=3e^{-5t+1}$

17)  $f(t)=\frac{t^2+1}{t^3+1}$

18) Find the first and second derivatives of the function  $f(t)=t^3+3t^2-9t+5$ .

a) When is the first derivative zero? On what intervals is it positive? ...negative?  
What does this tell you about those regions of the graph?

b) When is the second derivative zero? On what intervals is it positive? ...negative?  
What does this tell you about those regions of the graph?

c) Use this information to sketch a reasonable graph of  $f(t)$ .

