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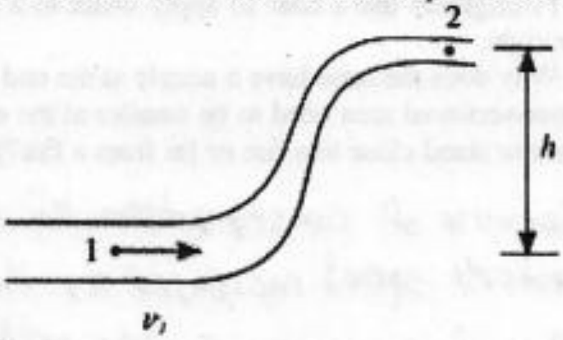
1. A fluid circuit has a pipe containing water which rises a height h, and the cross sectional area decreases to half its initial value as shown. The speed of the flow in the lower section is $v_1 = 10.0$ m/s and the pressure is $P_1 = 200$ kPa. Any dissipation can be ignored.
$\rho_w = 1000$ kg/m^3. Use $g = 10$ m/s^2.



a) At which point (point 1 or point 2) is the speed of the flow greatest and what is its value?

The pipe becomes narrower at 2, so the water's speed must increase.

Current is conserved: $I_1 = I_2$

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\cancel{A_1} v_1}{\frac{1}{2}\cancel{A_1}} = 2 \cdot v_1$$

$$\boxed{v_2 = 20\,\tfrac{m}{s}}$$

b) What is the greatest value h can have? Hint, the lowest value pressure can have it zero.

Suppose the pipe is flexible and we gradually raise the height of 2. Then the pressure at 2 will gradually decrease; when P_2 reaches 0 Pa, lifting 2 any higher will result in water not reaching the top.
So let's set $P_2 = 0$ Pa and solve for h_2. (We set $h_1 = 0$.)

$$\Delta P + \rho g \Delta h + \tfrac{1}{2}\rho \cdot \Delta(v^2) = 0 \quad \text{(no pump; negligible resistance)}$$

$$\overset{0}{\cancel{P_2}} - P_1 + \rho g (h_2 - \overset{0}{\cancel{h_1}}) + \tfrac{1}{2}\cdot\rho\cdot(v_2^2 - v_1^2) = 0$$

$$h_2 = \frac{P_1 - \frac{1}{2}\rho\cdot(v_2^2 - v_1^2)}{\rho \cdot g} = \frac{200\,000\,\text{Pa} - \frac{1}{2}\cdot 1000\,\frac{kg}{m^3}\cdot\left((20\,\frac{m}{s})^2 - (10\,\frac{m}{s})^2\right)}{1000\,\frac{kg}{m^3}\cdot 10\,\frac{m}{s^2}}$$

$$\boxed{h_2 = 5\,m}$$

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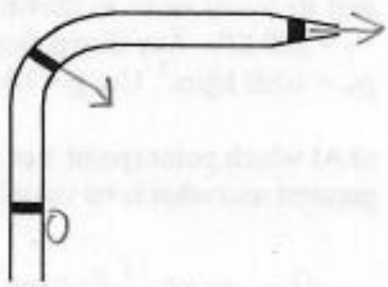
2. Firefighters use a hose to apply water to a fire. Such a hose is shown to the right.



a) Why does the hose have a nozzle at the end (in other words, why does the cross-sectional area need to be smaller at the end)? (Hint: does a firefighter want to stand close to a fire or far from a fire?)

Because of conservation of current, the water's speed increases as the hose narrows. The fast-moving water will travel further as it falls, allowing the firefighters to stand further from the fire.

b) A small amount of water (approximately 1 liter) is shown at three different locations as it moves through the hose. **On the picture**, draw a vector at each location showing the **direction of the net force** on this 1 liter of water when it is at that location. If the net force is zero then just write a 0 next to that location.

Straight section: no change in speed or direction, so $\vec{a}=\vec{0}$, so $\vec{F}_{net}=\vec{0}$.

Curved section: speed is constant, but direction is changing, so $\vec{F}_{centripetal}$ points towards center of circular path.

Nozzle: speed increases as water moves in a straight line, so $\vec{F}_{net}$ is in same direction as the water's motion.

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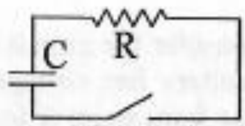
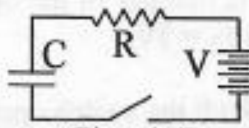
3. The figure to the right shows two circuits with a resistor with resistance *R* and capacitor with capacitance *C*, and a switch which has been left open. In circuit *A*, the capacitor is initially charged to a voltage *V* while in circuit *B* the capacitor is initially uncharged. Circuit *B* also contains a battery having voltage *V*. At time $t = 0$, the switch is suddenly closed. The graph below and to the right shows curves for the voltage across the capacitor V_C, as a function of time for different values of *V*, *R*, *C*.



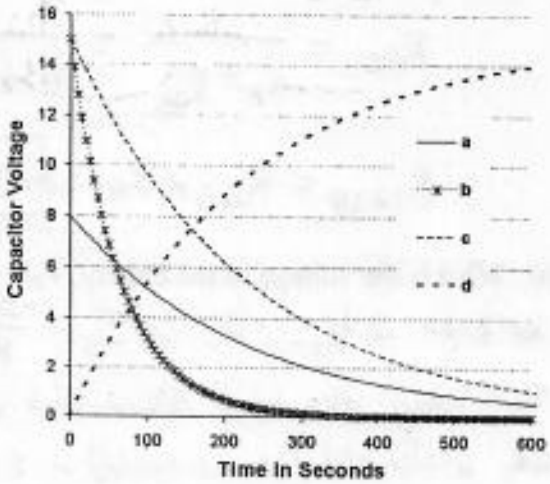
Circuit A



Circuit B

a) The table below shows the values of *V*, *R*, and *C*.and which circuit (*A* or *B*) produced the curves in the graph. In the space beside each of these, write the letter for the curve which it produced.

V (Volts)	*R* (Ohms)	**C** (Farads)	Circuit	Curve
15	8	8	*A*	b
15	15	15	*B*	d
8	15	15	*A*	a
15	15	15	*A*	c



b) Give a brief but clear explanation of how you decided which line in the chart produced curve *b*.

Curve b is discharging from 15 V, so it (along with curve c) could be row 1 or 4 of the data table. Note that curve b decays much more quickly than curve c, indicating that b has lower resistance (fast current) and lower capacitance (less stored charge to begin with). Curve b thus matches the first row.

c) Give a brief but clear explanation of how you decided which line in the chart produced curve *d*.

Curve d shows an increasing voltage, meaning the capacitor is charging rather than discharging – circuit B. The second row of the data table is the only one that matches.

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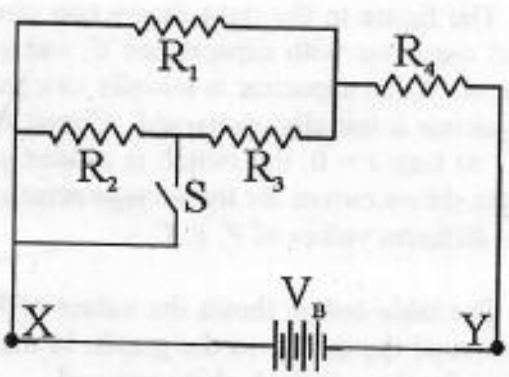
4. Consider the circuit shown in the figure to the right, in which the battery has voltage V_B and is connected across the resistor circuit from point X to point Y. The resistors have the following values: $R_1 = 32\Omega$, $R_2 = 20\Omega$, $R_3 = 12\Omega$, $R_4 = 24\Omega$. S is a switch, and is initially in the open position as shown. The voltage across R_3 is $V_3 = 3\text{V}$.



a) With the switch open (as drawn), find the equivalent resistance of all the resistors in this circuit.

$R_{23} = R_2 + R_3 = 32\Omega$ (series)

$R_{123} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_{23}}} = \frac{1}{\frac{1}{32}+\frac{1}{32}} = 16\Omega$ (parallel)

$R_{1234} = R_{123} + R_4 = \boxed{40\Omega}$ (series)

b) What is the voltage of the battery, V_B?

We know $\Delta V_{R_3} = -3V$, so $I_3 = \frac{-\Delta V_{R_3}}{R_3} = \frac{1}{4}A$. I_2 is also $\frac{1}{4}A$ (because series).

That means the total ΔV on that branch is $-I_2R_2 - I_3R_3 = -8V$.

ΔV_{R_1} is also $-8V$ (because parallel) so $I_1 = \frac{-\Delta V_{R_1}}{R_1} = \frac{1}{4}A$. Because of the junction rule, these currents combine to make $I_4 = \frac{1}{2}A$, so $\Delta V_{R_4} = -I_4R_4 = -12V$.

Consider the outer loop: $\mathcal{E} - I_1R_1 - I_4R_4 = 0$, so $\boxed{\mathcal{E} = 20V}$

c) If the switch S is closed, tell how each of the following quantities be affected and explain your reasoning:

i) The voltage across R_1?

read third

R_1, R_4, and the battery form a loop, so ΔV_{R_1} and ΔV_{R_4} together must add up to a constant. If R_4 is getting more voltage, then there is less voltage available for R_1.

ii) The current through R_2?

read first

drops to 0A, because there is now a resistanceless path (short circuit) connecting the two ends of R_2. That means $\Delta V = 0$, so no current flows through R_2

iii) The voltage across R_3?

read fourth

The lower branch is now getting the same reduced voltage as R_1 (because parallel), but with R_2 gone, R_3 no longer has to share, so it gets all that voltage, leading to more voltage for R_3.

iv) The voltage across R_4?.

read second

With R_2 effectively removed, the ~~total~~ R_{eq} decreases, so the battery—and therefore also R_4—gets more current. More current but same resistance means that R_4 now has a larger ΔV.

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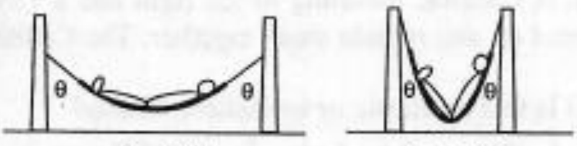
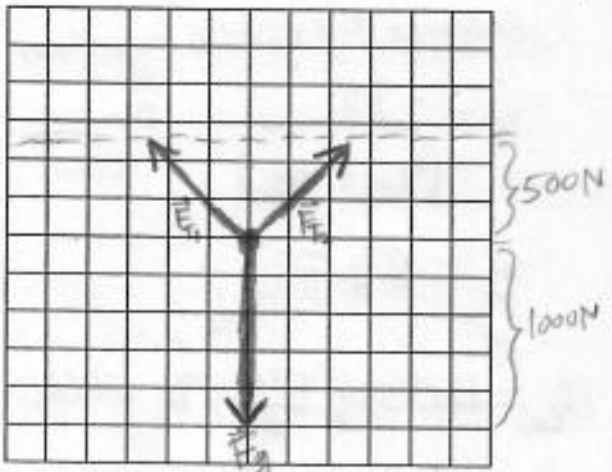
5. Two students (each with a mass of 100kg) are lying in hammocks (suspended between two posts) as shown in the pictures to the right (θ is the angle between the cords holding the hammock and the vertical direction). Consider a particular hammock plus the strings holding the hammock up plus the student in the hammock to be a **single object**. In this problem you can take the gravitational field to be g = 10 N/kg.



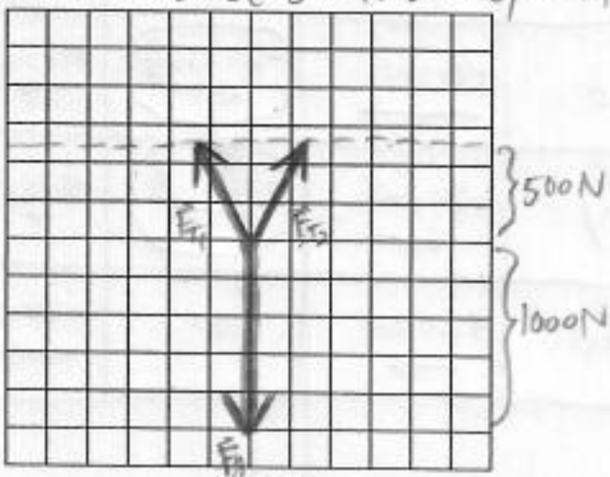
θ = 45°   θ = 25°

$\|\vec{F}_g\| = 1000N$

a) Draw an appropriately scaled and labeled force diagram for each of these two objects shown in the pictures (one force diagram in each grid below). Note that the y-component of each tension force must be 500 N, so they counter $\vec{F}_g$.



θ = 45°
scale: □ = 200N



θ = 25°

b) If the strings holding up the hammock could support any force (so they couldn't break) would it be possible to suspend the hammock so that the strings were both exactly horizontal (i.e. θ = 90°)? Explain.

No. The strings must provide a total y-component of +1000 N to counteract gravity; if they were truly horizontal, they would have no y-component. The strings must sag at least a little bit.

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6. A Cadillac traveling to the right and a Toyota traveling to the left with equal speeds of 20.0 m/s collide head on and remain stuck together. The Cadillac has a mass of 2000 kg and the Toyota has 1000 kg.

a) Is this an elastic or inelastic collision?

We could calculate KE_i and KE_f to be sure, but generally speaking if the objects stick together, then the collision is inelastic.

b) Find the change in momentum of each car.

	$\vec{p_i}$	$\Delta\vec{p}$	$\vec{p_f}$
Cad	$+40\,000 \rightarrow$	$-26\,666\frac{2}{3} \leftarrow$	$+13\,333\frac{1}{3} \rightarrow$
Toy	$-20\,000 \leftarrow$	$+26\,666\frac{2}{3} \rightarrow$	$+6666\frac{2}{3} \rightarrow$
total	$+20\,000 \rightarrow$	0	$+20\,000 \rightarrow$

$$p_f = (m_{Cad} + m_{Toy}) \cdot v_f$$

(because they stick together)

$$v_f = \frac{p_f}{(m_{Cad} + m_{Toy})} = \frac{20\,000}{3000}$$

$$v_f = 6\tfrac{2}{3}\ \frac{m}{s}$$

$$p_{f_{Cad}} = 13\,333\tfrac{1}{3}\ \frac{kg \cdot m}{s},\quad p_{f_{Toy}} = 6666\tfrac{2}{3}\ \frac{kg \cdot m}{s}$$

c) Find the change in velocity of each car.

$$\Delta v_{Cad} = 6\tfrac{2}{3}\ \frac{m}{s} - 20\ \frac{m}{s} = \boxed{-13\tfrac{1}{3}\ \frac{m}{s}}$$

$$\Delta v_{Toy} = 6\tfrac{2}{3}\ \frac{m}{s} - \left(-20\ \frac{m}{s}\right) = \boxed{+26\tfrac{2}{3}\ \frac{m}{s}}$$

d) Which car experiences the greatest average force?

Newton's Third Law explains that they experience equal and opposite forces ($\vec{F}_{A\,on\,B} = -\vec{F}_{B\,on\,A}$).

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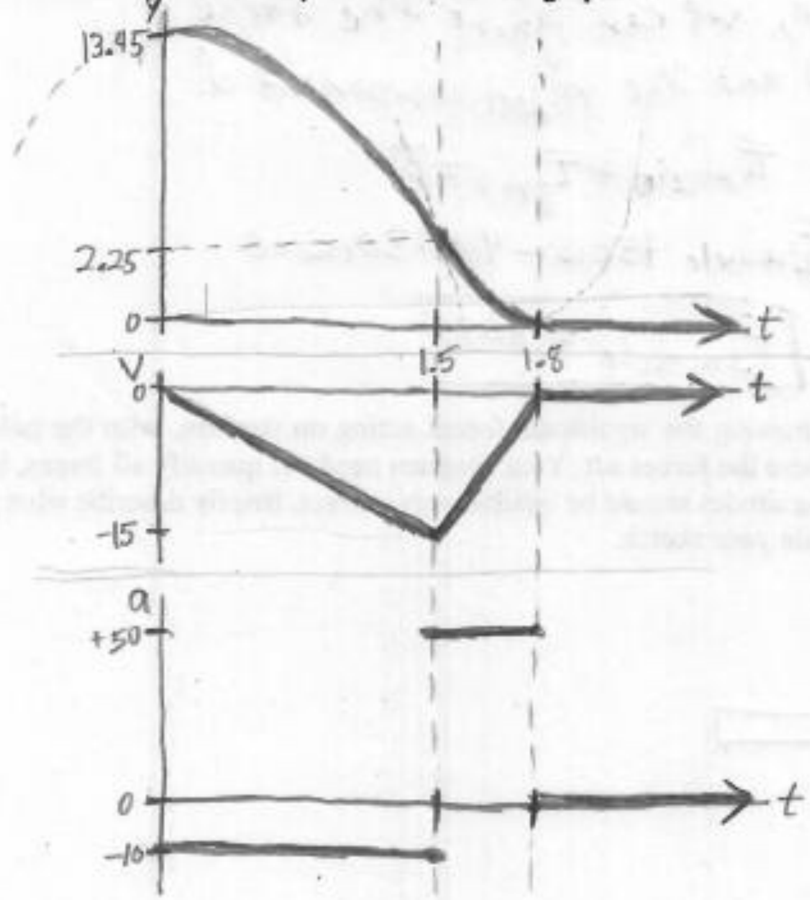
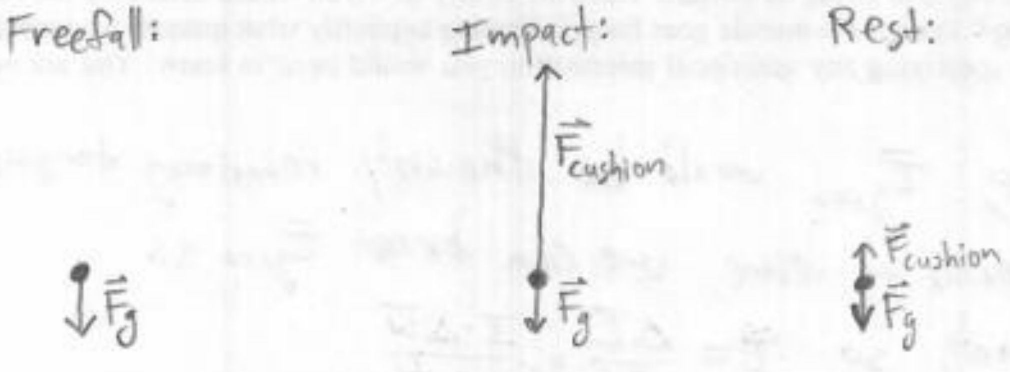
7. A person must flee a burning building through a third story window. Rescuers have positioned a large cushion below. The person drops off the window ledge, instinctively assuming a rather compact shape, and falls for 1.5s before contacting the cushion. The cushion exerts a nearly constant force and brings the person to rest in 0.30s.

a) Sketch three graphs, acceleration, velocity, and position, versus time from $t = 0$ s to $t = 1.8$ s. Attach *numerical* values to your velocity and acceleration graphs.



b) Sketch two properly labeled and *scaled* force diagrams for the person, one while falling before contacting the cushion, and the other while the cushion is bringing the person to rest.



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8. A man holds his arm horizontally outstretched. His deltoid muscle is attached to the arm at a point 15cm from the shoulder joint and pulls on the arm in the direction indicated.

The weight of his arm is 40N and its center of mass is 30cm from the shoulder joint.

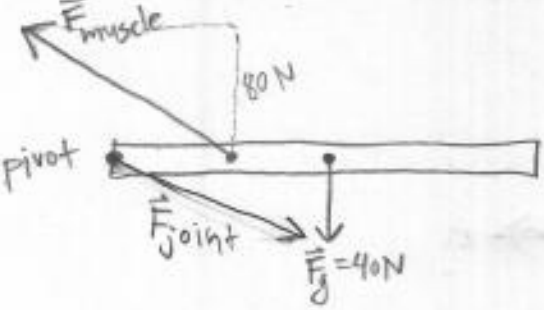
a) Determine the tangential component ($F_{tangential}$) of the force exerted by the muscle on the arm.

For purposes of torque, we can ignore the force from the joint ($r=0$) and the radial component of the muscle force. $\vec{\tau}_{muscle} + \vec{\tau}_{grav} = \vec{0}$

$$F_{\perp muscle} \cdot 15cm - 40N \cdot 30cm = 0$$

$$\boxed{F_{\perp muscle} = 80N}$$

b) Make an extended force diagram showing the significant forces acting on the arm, with the tails of the force vectors at the actual locations where the forces act. Your diagram need not quantify all forces, but their approximate directions and relative magnitudes should be qualitatively correct. Briefly describe what physics principle(s) you are applying to complete your sketch.



c) Suppose now that the person's muscle suddenly goes completely limp, exerting essentially no force on the arm. The person's arm would begin to swing downward. Describe clearly how you would determine the rate at which it would be swinging 0.1s after the muscle goes limp, indicating explicitly what quantity you would actually be determining and specifying any additional information you would need to know. You are not asked for numerical values.

In this case, $\vec{\tau}_{grav}$ would be the only remaining torque. For a short period of time, we can treat $\vec{\tau}_{grav}$ as almost constant, so $\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t} = \frac{I \cdot \Delta \vec{\omega}}{\Delta t}$,

$$\text{so } \Delta \vec{\omega} = \frac{\vec{\tau}_{grav} \cdot \Delta t}{I_{arm}} = \frac{40N \cdot 30cm \cdot 0.1s}{I_{arm}}$$

We would still need to know I_{arm}.