<u>Extra Practice: Conservation of Energy</u>

(see next page for any material property values you may need)

1) A team of dwarven mining engineers needs to seal off a tunnel in a hurry (there's a balrog on its way up from the depths), so they decide to form a plug by flooding it first with basaltic magma at 1300°C and then with water at 10°C. The magma solidifies; the steam is allowed to leave through ventilation shafts as it forms, so it doesn't get any warmer after it vaporizes. If the desired end result is 1,000,000 kg of solid basalt at 500°C, how much water is needed? (Assume that the air, the tunnel walls, and the balrog do not absorb or contribute a significant amount of heat.)

2) Some solar power plants are experimenting with the use of PCMs (phase-change materials, such as the sodium acetate you worked with in lab) as a method of *storing* extra energy after bright sunny days. When the power plant is producing more energy than is needed at the moment, excess energy is used to melt a quantity of PCM; on cloudy days when not enough power is generated, the molten PCM is allowed to solidify and the heat it gives off is used to generate extra electrical power.

   a) If a material is particularly useful for storing energy in this way, does this tell you something about value of the c, the ΔH, or the T_{melt} of the material? Would you expect that value to be large or small? Why?

   b) Suppose that a certain city requires 1.21 GW of power (that is, 1.21 billion joules per second), and the Department of Power wants to make sure that it can provide power reliably throughout up to 24 hours of complete darkness by using salt as a PCM. How much salt should the power plant have in store to be prepared for this?

3) Suppose we have a 3 kg block of solid iron at 500°C. We place this block of iron into an insulated vat of icy water that is currently half solid and half liquid, close the lid, and wait for the system to reach thermal equilibrium.

   a) If the vat starts with 1000 kg of icy water, find the final equilibrium temperature and the final state of the water.

   b) If the vat starts with 0.01 kg of icy water, find the final equilibrium temperature and the final state of the water. (Of course here it's more like we're dripping the icy water onto the iron instead of placing the iron in the water.)

   c) If the vat starts with 1 kg of icy water, find the final equilibrium temperature and the final state of the water.

Useful Information

Material	c_{solid} (kJ/kgK)	c_{liquid} (kJ/kgK)	c_{gas} (kJ/kgK)	ΔH_{melt} (kJ/kg)	ΔH_{boil} (kJ/kg)	T_{melt} (°C)	T_{boil} (°C)
basalt	1.4	1.0	—	400	—	1200	2222
water	2.06	4.18	2.92	333.5	2257	0	100
iron	0.45	—	—	247.3	—	1538	—
salt	—	—	—	410	—	385	—

(Some values have been left out because are not relevant for these problems.)

Relevant Videos

Molten rock mixing with liquid water:
`https://www.youtube.com/watch?v=ahZD95l1MvM`

Hot nickel ball placed on ice:
`https://www.youtube.com/watch?v=w0o5xVkzo54`

Solutions

1) The granite starts above its melting point and ends below its melting point, so it will need to undergo a change in E_{th}, a change in E_{bond}, and another change in E_{th}. The water starts below its boiling point but is vented away as soon as it vaporizes, so it will need only a change in E_{th} (to reach 100°C) and a change in E_{bond}. That means our conservation of energy equation will look like this:

$$\Delta E_{th\,magma} + \Delta E_{bond\,magma/basalt} + \Delta E_{th\,basalt} + \Delta E_{th\,water} + \Delta E_{bond\,water/steam} = 0$$

Substituting the relevant formulas:

$$mc\Delta T_{magma} + \Delta m\Delta H_{magma} + mc\Delta T_{basalt} + mc\Delta T_{water} + \Delta m\Delta H_{water} = 0$$

$$mc(T_f - T_i)_{magma} + (m_f - m_i)\Delta H_{magma} + mc(T_f - T_i)_{basalt} + mc(T_f - T_i)_{water} + (m_f - m_i)\Delta H_{water} = 0$$

Recall that we uses the mass of the *higher* energy state (liquid basalt; gaseous water) as the indicator for bond energy:

$$mc(T_f - T_i)_{magma} + (0 - m)\Delta H_{magma} + mc(T_f - T_i)_{basalt} + mc(T_f - T_i)_{water} + (m - 0)\Delta H_{water} = 0$$

$$mc(T_f - T_i)_{magma} + (-m)\Delta H_{magma} + mc(T_f - T_i)_{basalt} + mc(T_f - T_i)_{water} + m\Delta H_{water} = 0$$

Of these, the only unknown value is the mass of the water, so isolate m_{water}:

$$m_{water} = \frac{-mc(T_f - T_i)_{magma} + m\Delta H_{magma} - mc(T_f - T_i)_{basalt}}{c(T_f - T_i)_{water} + \Delta H_{water}}$$

We can factor out the m from the numerator, because they all mean 10^6 kg:

$$m_{water} = \frac{-10^9\,\text{kg}\cdot\left(1\,\text{kJ/kgK}\cdot(1200\,^\circ\text{C} - 1300\,^\circ\text{C}) + 400\,\text{kJ/kg} - 1.4\,\text{kj/kgK}\cdot(500\,^\circ\text{C} - 1200\,^\circ\text{C})\right)}{4.18\,\text{kJ/kgK}\cdot(100\,^\circ\text{C} - 10\,^\circ\text{C}) + 2257\,\text{kJ/kg}}$$

$m_{water} = 562{,}000{,}000\,\text{kg}$ or 562,000 metric tons.

(This means that cooling off each kg of magma requires 562 kg of water!)

That balrog won't know what him, but the reservoir might be a little low for a while.

2) a) If we are storing energy by pushing the PCM through a phase change, what we're storing is ΔE_{bond}, that is, $\Delta m\Delta H$. Ideally we want the PCM to store as much energy per kilogram as possible—and energy per kilogram (of phase-changed material) is precisely the definition of ΔH, so a <u>large</u> value of ΔH is what we're looking for.

b) Power means energy per unit of time, so to find the energy required we multiply power by time: 1.21 GW (1.21×10^9 joules per second) times 24 hours ($24\times60\times60$ seconds) equals 1.05×10^{14} J of E_{bond}, so that's the amount of bond energy the salt needs to store. The formula for ΔE_{bond} is

$\Delta E_{bond} = \Delta m\Delta H$, so $\Delta m = \Delta E_{bond} / \Delta H = (1.05\times10^{14}\text{ J}) / (4.1\times10^5\text{ J/kg}) \approx 2.56\times10^8$ kg.

In other words, about 256,000 metric tons. That's a lot of salt! Energy storage on this large a scale is expensive... but then again the long-term costs of more traditional means of generating power are far far worse. Little by little we progress!

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3) a) With that much water, we can assume that the phase change never finishes—that is, there will still be some solid ice at the end. Thus we can assume that the final equilibrium temperature is still 0°C, and the only question is how much ice melts as the iron cools down. The conservation of energy equation will look like this:

$$\Delta E_{th\,iron} + \Delta E_{bond\,ice/water} = 0$$

$$mc\Delta T_{iron} + \Delta m \Delta H_{water} = 0$$

$$\Delta m = -mc\Delta T_{iron} / \Delta H_{water} = -\ 3\text{kg} \cdot 0.45\ \text{kJ/kgK} \cdot (-500\text{K}) / (333.5\ \text{kJ/K}) = 2.02\ \text{kg}$$

The fact that the result is positive means we are gaining 2.02 kg of liquid.

b) With so little water, we can assume that the iron hardly changes temperature at all, and that the icy water completely melts, completely boils, and reaches equilibrium with the still very hot iron. The equation will look like this:

$$\Delta E_{th\,iron} + \Delta E_{bond\,ice/water} + \Delta E_{th\,water} + \Delta E_{bond\,water/steam} + \Delta E_{th\,steam} = 0$$

$$mc\Delta T_{iron} + \Delta m\Delta H_{water} + mc\Delta T_{water} + \Delta m \Delta H_{steam} + mc\Delta T_{steam} = 0$$

$$mc(T_f - T_i)_{iron} + (m_f - m_i)\Delta H_{water} + mc(T_f - T_i)_{water} + (m_f - m_i)\Delta H_{steam} + mc(T_f - T_i)_{steam} = 0$$

$$mc(T_f - T_i)_{iron} + (m_f - m_i)\Delta H_{water} + mc(T_f - T_i)_{water} + (m_f - m_i)\Delta H_{steam} + mc(T_f - T_i)_{steam} = 0$$

$$3\text{kg} \cdot .45\text{kJ/kgK} \cdot (T_f - 500°\text{C}) + .005\text{kg} \cdot 333.5\text{kJ/kg}$$
$$+ .01\text{kg} \cdot 4.18\text{kJ/kgK} \cdot 100\text{K} + .01\text{kg} \cdot 2257\text{kJ/kg}$$
$$+ .01\text{kg} \cdot 2.92\text{kJ/kgK} \cdot (T_f - 100°\text{C}) = 0$$

The only unknown left is T_f, so we can solve for that final temperature.

I'm getting 471°C, so yes, the water boils, but the iron cools off quite a bit.

(I wasn't expecting that, but I should have—phase changes take a lot of energy, especially with water's large ΔH, and water also has a much higher c than iron!)

c) This one is trickier because it is not immediately obvious whether the hot iron or cold icy water will dominate, since their masses are not extremely different. Instead we have to take it one step at a time, checking which object will reach any given critical point first. For example, to start with, either the water will completely melt first (as in b) or the iron will cool all the way to 0°C first (as in a). To determine which one happens, we find the amount of ΔE each change involves.

Suppose, for instance, that the water completely melts. That's ΔE_{bond}, so

$$\Delta E_{bond\,ice/water} = \Delta m \Delta H_{water} = .5\text{kg} \cdot 333.5\text{kJ/kg} = 166.75\ \text{kJ} \approx 167\ \text{kJ}$$

is the amount of energy required to melt all the ice.

Alternatively, suppose the iron drops all the way to 0°C. That's ΔE_{th}, so

$$\Delta E_{th\,iron} = mc\Delta T = 3\text{kg} \cdot .45\text{kJ/kgK} \cdot (-500\text{K}) = -675\ \text{kJ}$$

is the amount of energy the iron could give off as it cools to 0°C.

The ΔE for the ice melting is much smaller than the ΔE for the iron cooling, so we know that the ice melts completely long before the iron would reach 0°C.

What happens next, though? Does the water reach 100°C and start boiling while the iron is still hot? ...or does the iron cool beyond 100°C and reach a thermal equilibrium with the still-liquid water? To find out, investigate those transitions:

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Suppose the water reaches 100°C. That's ΔE_{th}, so

$\Delta E_{th\,water} = mc\Delta T$ = 1kg · 4.18kJ/kgK · 100K = 418 kJ

is the amount of energy the water must gain to reach 100°C (barely).

We'll have to combine this with the energy it took to melt the ice in the first place, for a total energy gain of 167 kJ + 418 kJ = 585 kJ.

Alternatively, suppose the iron drops from 500°C to 100°C. That's ΔE_{th}, so

$\Delta E_{th\,iron} = mc\Delta T$ = 3kg · .45kJ/kgK · (-400K) = -540 kJ

is the energy the iron is able to provide while it cools to 100°C.

Now we're getting somewhere—this means the iron can't provide enough energy to melt the icy water AND raise its temperature to 100°C! That tells us that the iron drops all the way to 100°C while the now-liquid water is still on its way up towards 100°C, so they will meet somewhere in the middle, with the water all liquid.

With that knowledge, we can finally set up a correct energy conservation equation:

$\Delta E_{bond\,ice/water} + \Delta E_{th\,water} + \Delta E_{th\,iron} = 0$

$\Delta m\Delta H_{water} + mc\Delta T_{water} + mc\Delta T_{iron} = 0$

.5kg · 333.5kJ/kg + 1kg · 4.18kJ/kgK · (T_f – 0°C)

+ 3kg · .45kJ/kgK · (T_f – 500°C) = 0

The only unknown left is T_f, so we can solve for that final temperature.

I'm getting about 92°C, which is consistent with the notion that the iron doesn't have quite enough energy to bring the water up to boiling.