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Math 222: Midterm #3

Show all work on problems that are more than straightforward calculations. Clearly mark answers.

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1) Use the integral definition (not the Laplace table) to find $\mathcal{L}\{\, 4\, t\, e^{t} \,\}$. Show all work.

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2) Prove that $\mathcal{L}$ is *linear*; that is, show *why* sums can be split up and constants can be pulled out.

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3) Explain why a $\delta(t-a)$ in the forcing function leads to a $u(t-a)$ in the solution.
(There is a mathematical explanation AND a physical explanation—choose one or try for both!)

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4) Use the Laplace transform to solve: $y''-2y'+2y=5e^{-t}$; $y(0)=0$, $y'(0)=1$.

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5) Solve for y: $2y''+8y'+16y=3\delta(t-5\pi)$; $y(0)=1$, $y'(0)=0$.
Also: what physical situation might this represent?

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6) Solve for y: $y''+y=\begin{cases} 2 & \text{if } 3<t<6 \\ 1 & \text{elsewhere} \end{cases}$; $y(0)=0$, $y'(0)=0$. (Hint: use unit step functions.)

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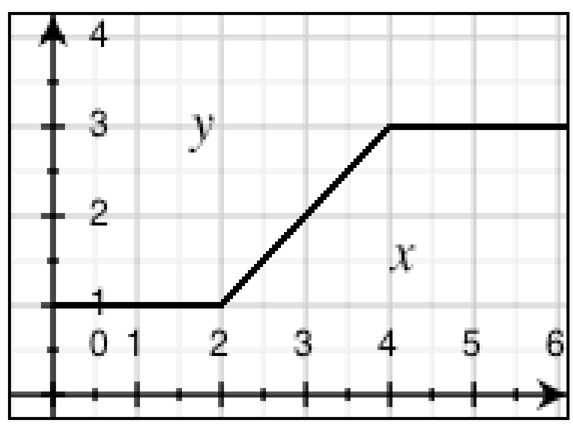
7) The graph of $y=f(t)$ is shown here.



a) Write an equation for $f(t)$ in terms of u_c.

b) Find the Laplace transform of $f(t)$.

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BONUS ROUND!

XC1) Determine the Laplace transform of t^n from the integral definition, where *n* is an integer.

XC2) Suppose an underdamped mass-on-a-spring is oscillating, and you have the opportunity to kick it at a certain time (say, $t = 6\pi$) while it is crossing the equilibrium point. Describe in detail how you would calculate *what impulse* your kick would need to have in order to *stop* the motion entirely.

XC3) So far we've been using the method of Laplace transforms to find *specific* solutions of initial value problems. How could it be used to find instead a *general* solution (with unknown c_1, c_2, etc)?

XC4) Tell me an interesting fact or story about Pierre-Simon Laplace, Oliver Heaviside, or Paul Dirac.