 Name:________________________________

Math 222: Midterm #3

Show all work on problems that are more than straightforward calculations. Clearly mark answers.
All solutions should be written with pure real numbers if at all possible.

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1) Consider the third-order differential equation $ay'''+by''+cy'+dy=0$.

a) Rewrite this problem as a system of first-order differential equations for $x_1(t), x_2(t), x_3(t)$.

b) Rewrite that system as a single first-order matrix equation for the vector $\vec{x}(t)$. Do not solve.

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2) Determine if the vectors $\vec{x}_1=\begin{bmatrix}1\\2\\3\end{bmatrix}, \vec{x}_2=\begin{bmatrix}4\\5\\6\end{bmatrix}, \vec{x}_3=\begin{bmatrix}7\\8\\9\end{bmatrix}$ are linearly independent. How can you tell?

If they are linearly dependent instead, find a linear combination of them that equals $\vec{0}$.

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3) Find the specific solution for $\vec{x}$: $\vec{x}\,' = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \vec{x}, \quad \vec{x}(0) = \begin{bmatrix} -4 \\ 2 \end{bmatrix}.$

4) Find the general solution for $\vec{x}$: $\vec{x}\,' = \begin{bmatrix} 2 & 8 \\ -2 & 2 \end{bmatrix} \vec{x}$

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5) Find the general solution for $\vec{x}$: $\vec{x}\,' = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix} \vec{x}$

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BONUS ROUND!

XC1) Find the eigenvalues and eigenvectors of $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

XC2) Prove that a set of vectors is linearly independent if their Wronskian is ______. (fill in the blank)

XC3) Write a 4x4 matrix that has some nonreal entries, but whose eigenvalues are all real.
Explain how you know that they will all be real. How many independent eigenvectors has it?

XC4) If you've made it to here and still have some time left, try graphing the solutions to problems 3, 4, and 5 on a calculator. You won't get any points for it, but I bet they look pretty awesome, and in a way, isn't making awesome graphs what differential equations is really all about?