 Name:_____

Math 222: Midterm #2

Show all work on problems that are more than straightforward calculations. Clearly mark answers.

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1) Consider a mass-on-a-spring system with the given mass, damping coefficient, and spring constant. For each situation, **determine** whether the system is underdamped, overdamped, or critically damped; **explain** how you can tell; and **sketch** a sample graph of what its motion might look like. Do not solve.

A) $m=3$, $b=6$, $k=3$

B) $m=1$, $b=2$, $k=3$

C) $m=2$, $b=5$, $k=3$

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2) Suppose an electric circuit consists of a resistor (with R=20 Ω), an inductor (with L=10 H), a capacitor (with C = 0.001 F), and a voltage source providing oscillating voltage E=60sin(120π t), all connected in series (i.e. as a single loop). Write a differential equation to model this circuit, using t (time) as the independent variable and q (charge) as the dependent variable. Do not solve.

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3) Find the specific solution to $y''-2y'-15=0$. ith initial conditions $y(0)=1, y'(0)=-1$.

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4) Find the general solution to $y''+4y'+13y=0$.

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5) Find the specific solution to $y''+4y'+13y=26t^2$ with initial conditions $y(0)=0, y'(0)=1$.

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6) Assume that $y_1=e^{5t}$ and $y_2=te^{5t}$ are solutions to some second order differential equation.

a) Calculate the Wronskian $W(y_1, y_2)$.

b) What does the value of $W(y_1, y_2)$ tell you about y_1 and y_2?

7) Find the general solution to $y''-10y'+25y=0$.

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8) Find the general solution to $y'' - 10y' + 25y = \frac{e^{5t}}{t^5}$.

9) Find the general solution to $y'''' + y''' - 3y'' - 5y' - 2y = 0$.

Math 222 Extra Credit

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Show me what you can do!

Bonus I: Find the 8th derivative of $e^x \sin(x)$. There *is* a shortcut!
Don't waste time trying to find ALL of the lesser derivatives. Look for patterns.

Bonus II: Prove that the difference of any two solutions of any nonhomogeneous equation must be a solution of the corresponding homogeneous equation.

Bonus III: Explain *why* solving the characteristic (aka auxiliary) quadratic leads to the correct *r*-values.

Bonus IV: You know how to raise *e* to complex powers, but how would you calculate, say, 5^{2+3i}?

Bonus V: Find the quasi-frequency of the underdamped system in Problem #1.