

Math 222: Midterm #2

Show all work on problems that are more than straightforward calculations. Clearly mark answers.

1) A 1.5 kg mass hangs on a spring with spring constant $6 \frac{\text{kg}}{\text{s}^2}$ and damping coefficient $6 \frac{\text{N s}}{\text{m}}$. The mass is pulled down 0.5 m and released. (Don't worry about the units; they work out.)

a) Write an initial value problem to model this situation.

b) Is this system underdamped, overdamped, or critically damped? How can you tell?

2) a) Find the general solution for $y: y''' - y'' - y' + y = 0$.

b) Find the specific solution for the initial conditions $y(0)=0, y'(0)=1, y''(0)=2$.

3) Determine the Laplace transform of $f(t) = 3t^2$ from the integral definition. Show all work.

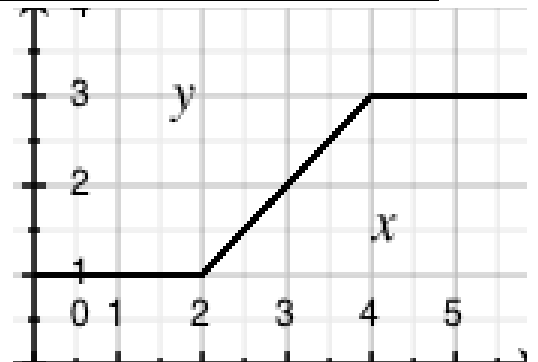
4) Use the Laplace transform to solve: $y'' - 2y' + 2y = 5e^{-t}$; $y(0) = 0$, $y'(0) = 1$.

5) Solve for y : $2y'' + 8y' + 16y = 3\delta(t-5)$; $y(0)=0$, $y'(0)=0$.

6) Solve for y : $y'' + y = \begin{cases} 1 & \text{if } 5 \leq t < 10 \\ 0 & \text{elsewhere} \end{cases}$; $y(0)=0$, $y'(0)=0$.

7) The graph of $y=f(t)$ is shown here.

a) Write an equation for $f(t)$ in terms of u_c .



b) Find the Laplace transform of $f(t)$.

BONUS ROUND!

XC1) Determine the Laplace transform of t^n from the integral definition, where n is an integer.

XC2) Prove that the Laplace transform of $\delta(t-c)$ is e^{-cs} .

XC3) Find all values, including nonreal values, of $\sqrt[5]{32}$. In other words, solve $x^5=32$ for x .

XC4) Write a brief description of a physical situation that could be modelled by the equation in #5.

XC5) What is the quasi-frequency of the solution to #5?