 Name:________________________________

Math 222: Midterm #1

Show all work on problems that are more than straightforward calculations. Clearly mark answers.

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1) Give an example of a differential equation that is 3rd order and nonlinear, and explain why it is.

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2) For the differential equation $x^2 y' + 4y^2 = 9$...

a) ...find all equilibrium solutions.

b) ...find the equations of a few other isoclines. What shapes do they form?

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3) Draw a direction field for $y' = 2 - \frac{y}{2}$. Sketch a solution curve going through the origin.

Does it exhibit *stable* equilibrium, *unstable* equilibrium, or *no* equilibrium? Where? How can you tell?

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4) Find the specific solution of $y' = x^2 e^{2x} + 2y$ passing through the point $(3, e^6)$.

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5) Find the general solution of $y' = \frac{2x}{x^2 y + y}$. (Hint: factor the denominator.)

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6) Find the specific solution to $(yx^2 + x) \cdot y' + xy^2 + y = 0$ passing through the point (1,2).
You may leave the solution in implicit form.

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7) A 200 gallon tank is initially **half-full** of salty water, with 50 grams of salt per gallon of water. Pure fresh water is poured into the tank at a rate of 10 gallons per hour. The well-stirred mixture drains out of the tank at 5 gallons per hour.

a) Set up an initial value problem to model this situation. Use $S(t)$ for the total amount of salt in the tank after t hours have passed.

b) Solve it to find $S(t)$.

c) What is the salt *concentration* (grams/gallon) of the water in the tank at the moment it overflows?

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8) Suppose a warehouse has a time constant of 3 hours, and the temperature outside varies sinusoidally with the extreme temperatures being 35°C each noon and 15°C each midnight. Write, but do not solve, a differential equation describing T(t), the temperature inside the warehouse.

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9) If $y' = 3 + x^2 - y$ and $y(0) = 4$, approximate $y(2)$ using Euler's method with **four steps.** Leave answer as a fraction. **Show all work.**

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Math 222 Extra Credit

Show me what you can do!

Bonus I: In problem #7, suppose that instead of fresh water, salty water was flowing in, with a *changing* salt concentration of $50\sin\left(\frac{2\pi}{12.4}t\right)+50$. Set up a differential equation to model this new situation and describe in detail what method could be used to solve it. What real-world situation could give rise to this kind of sinusoidally changing salinity? From where might this water be pumped?

Bonus II: Re-do problem #9 using 100 steps instead of four steps. A program will be useful here!
OR: write out how you would set up a spreadsheet (with formulas) to do this for you.

Bonus III: A cup of coffee is served at 95°C, and has cooled to 80°C five minutes later. The room's temperature is a constant 21°C. Find the time constant for the cup of coffee.

Bonus IV: Find the general solution of $y' = x^2 e^{2x} + y$.