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Math 222: Midterm #1

Show all work on problems that are more than straightforward calculations. Clearly mark answers.

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1) Explain what is meant by "3rd order nonlinear ordinary differential equation" and give an example.

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2) Find all equilibrium solutions of $y'' + t^2 y' - 4y^2 + 3 = 0$.

3) Does $y' = 6 - 2y$ exhibit stable equilibrium, unstable equilibrium, or no equilibrium at all?

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4) Find a general solution for $y' = x^2 e^{2x} + 2y$.

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5) Find the specific solution of $y' = \frac{2x}{x^2 y + y}$ passing through the point (0, –2).

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6) Find a general solution to $y' = -\frac{xy^2 + y}{yx^2 + x}$. You may leave the solution in implicit form.

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7) A tank initially holds 100 gallons of salty water, with 50 grams of salt per gallon of water. Pure fresh water is poured into the tank at a rate of 5 gallons per hour. The well-stirred mixture drains out of the tank at 10 gallons per hour.

a) Set up a differential equation to model this situation. Use $S(t)$ for the total amount of salt in the tank after t hours have passed.

b) Find $S(t)$.

c) What is the salt *concentration* (grams/gallon) of the water in the tank 1 minute before it's empty?

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8) If $y' = \frac{4 - ty}{1 + y^2}$ and $y(0) = -2$, approximate $y(1)$ using Euler's method and a t-step of $h = \frac{1}{2}$. Leave answer as a fraction.

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9) Assume that $y_1 = e^t \sin(t)$ and $y_2 = e^t \cos(t)$ are solutions to some second order differential equation.

a) Calculate the Wronskian $W(y_1, y_2)$.

b) What does this tell you about y_1 and y_2?

c) If this is enough information, what is the general solution to this differential equation? If not, what other information is needed?

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10) Find the general solution of $4y'' + 9y = 0$.

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11) Find the general solution of $4y'' + 9y = 3t^2$.

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12) Find the *specific* solution of $y''-10y'+25y=0$ with initial conditions $y(0) = 0,\ y'(0) = 8$.

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13) Find the general solution of $y''-10y'+25y=\frac{e^{5t}}{t^5}$.

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Math 222 Extra Credit

Show me what you can do!

Bonus I: Find the sixth derivative of $e^x \sin(x)$ (with respect to x). There *is* a shortcut.

Bonus II: If $y' = \frac{4 - ty}{1 + y^2}$ and $y(0) = -2$, find a *much closer* approximation of $y(1)$ with $h = 0.01$.
OR: if you don't have your program with you, describe (e.g., in pseudo-code) how it works.

Bonus III: Prove that the difference of any two solutions of any nonhomogeneous equation must be a solution of the corresponding homogeneous equation.

Bonus IV: Explain *why* solving the characteristic quadratic leads to the correct r-values.