 Name:________________________________

Math 222: Final Exam!

This is it! One last test and then you can go out into the world and do stuff with what you've learned! Show all work, clearly mark answers, etc., you know the drill. Now get started! You can do it!

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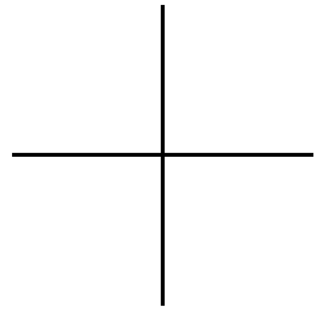
1) Draw a direction field of $y' = y^2 - 4$ and find all equilibrium solutions.
Are they stable, unstable, or neither? How can you tell?



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2) a) Find the general solution to $t^2 y' + 2ty = \cos t$. (It's linear, so integrating factors should work.)

b) Find the specific solution that passes through the point (π, 3).

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3) Let $y' = x^2 + y^2$, $y(0) = 0$. Use Euler's Method to approximate $y(1)$, using an x-step of $h = 0.25$.
At every step, be as accurate as your calculator will allow. Show x- and y-values at each step.

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4) Find the general solution to $y'''' - 16y = 0$. (Yes, that's the *fourth* derivative.)

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5) Find the general solution for $\vec{x}$: $\vec{x}\,' = \begin{bmatrix} 5 & -3 \\ 3 & -1 \end{bmatrix} \vec{x}$

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Problems 6, 7, and 8 are all about a mass-on-a-spring system as described here:
Assume that a **mass of 1** is attached to a spring with **spring constant 6** and **damping coefficient 9**.

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6) The mass is pulled 2 units away from equilibrium and released at $t = 0$. Assume no external force.
   a) Is this system underdamped, critically damped, or overdamped? How can you tell?

   b) Find an equation (specific solution) describing its motion y as a function of time t.

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7) Suppose instead that a steadily increasing external force, $F_{ext}(t) = 2t$, is applied.
   Find the *general* solution describing *all* possible paths the mass could take in this case.
   (For this problem, use a method from Chapter 4, *not* Laplace transforms.)

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Keep going! You're almost there!

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8) Suppose that the mass starts at rest at $y = 2$ again, but at time $t = 5$, it is struck by a hammer with an impulse of magnitude 10. Use Laplace transforms (and ∂) to find the equation of its motion.

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9) Suppose a warehouse has a time constant of 2 hours, and the temperature outside varies sinusoidally with the extreme temperatures being 30°C each noon and 10°C each midnight. Write, but do not solve, a differential equation describing T(t), the internal temperature. Assume $t = 0$ at the first midnight.

BONUS: Solve it anyway! You can do it!

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BONUS ROUND!

(You may need more paper. Please at least write the answers on this page.)

XC1) Find the eigenvalues and eigenvectors of $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

XC2) Prove that the difference of any two solutions of any nonhomogeneous equation must be a solution of the corresponding homogeneous equation. (I liked this question, so here it is again.)

XC3) One last fluid mixture problem before you go, because they're so much fun!
At time zero, Tank A holds 100 gallons of pure water and Tank B holds 50 gallons of pure orange juice.
Pure water is poured into Tank A at 3 gal/min and the mixture drains out the bottom at 4 gal/min.
Pure orange juice is poured into Tank B at 4 gal/min and the mixture drains out the bottom at 3 gal/min.
The tanks are linked with pipes: mixture flows from A to B at 5 gal/min, and from B to A at 6 gal/min.
Notice that the total volume of each tank remains constant. (Draw a diagram; you'll see what I mean.)

a) Set up a *system* of differential equations to model this situation.
Use A(t) and B(t) for the current amount of **juice** in each tank.

b) Solve for A(t) and B(t). Keep in mind the initial conditions!

c) After "a very long time," how much juice vs. water will be in each tank?

XC4) Explain in detail how differential equations can be used in your intended major and/or career.