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Math 222: Final Exam!

This is it! One last test and then you can go out into the world and do stuff with what you've learned! Show all work, clearly mark answers, etc., you know the drill. Now get started! You can do it!

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1) Find the equilibrium solution of $y' = 3y + 15$ and state whether it is stable, unstable, or neither.

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2) a) Find the general solution to $t^2 y' + 2ty = \cos t$. (It's linear, so integrating factors should work.)

b) Find the specific solution that passes through the point $(\pi, 3)$.

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3) Let $y' = t^2 + y^2$, $y(0) = 0$. Use Euler's Method to approximate $y(0.5)$, using a t-step of $h = 0.1$. At every step, be as accurate as your calculator will allow. Show t and y at each step.

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4) Find the general solution to the homogeneous equation $y''+4y'+5y=0$.

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5) Find the general solution to the nonhomogeneous equation $y''+4y'+5y=t+e^t$.

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6) Find the general solution to $y''''-16y=0$. (Yes, that's the *fourth* derivative.)

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Keep going! You're almost there!

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7) Find the specific solution for y: $2y''+8y=\begin{cases} 0 \text{ if } t<5 \\ 4 \text{ if } t>5 \end{cases}$; $y(0)=1$, $y'(0)=0$.

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8) Find the specific solution for $\vec{x}$: $\vec{x}'=\begin{bmatrix} 5 & -3 \\ 3 & -1 \end{bmatrix}\vec{x}$; $\vec{x}(0)=\begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

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BONUS ROUND!

(You will probably need more paper.)

XC1) Usually the Laplace transform is used to find the *specific* solution of an initial value problem. Describe how it could instead be used to find the *general* solution of a differential equation.

XC2) Prove that the difference of any two solutions of any nonhomogeneous equation must be a solution of the corresponding homogeneous equation. (I liked this question, so here it is again.)

XC3) Explain why the n complex nth roots of 1 must be equally spaced on a circle around the origin.

XC4) One last fluid mixture problem before you go, because they're so much fun:
At time zero, Tank A holds 100 gallons of pure water and Tank B holds 50 gallons of pure orange juice.
Pure water is poured into Tank A at 3 gal/min and the mixture drains out the bottom at 4 gal/min.
Pure orange juice is poured into Tank B at 4 gal/min and the mixture drains out the bottom at 3 gal/min.
The tanks are linked with pipes: mixture flows from A to B at 5 gal/min, and from B to A at 6 gal/min.
Notice that the total volume of each tank remains constant. (Draw a diagram; you'll see what I mean.)

a) Set up an initial value problem to model the situation. Use A(t) and B(t) for the current amount of juice in each tank (amount of water can be written in terms of A and B; no new variables needed).

b) Solve for A(t) and B(t). Keep in mind the initial conditions!

c) After "a very long time," how much juice vs. water will be in each tank?