Casey's Notecard on Quantum Notation

Properties of the eigenfunctions: (for the quantum harmonic oscillator; other systems will have different eigenfunctions)

- Hermite Substitutions:

multiplication by x:

$$x \cdot \psi_n = \sqrt{\frac{n}{2}} \cdot \psi_{n-1} + \sqrt{\frac{n+1}{2}} \cdot \psi_{n+1}$$

derivative with respect to x:

$$\frac{d}{dx} \psi_n = \sqrt{\frac{n}{2}} \cdot \psi_{n-1} - \sqrt{\frac{n+1}{2}} \cdot \psi_{n+1}$$

- Orthonormality:

$(\psi_a, \psi_b) = \delta_{a,b}$ (Kronecker delta: 1 if same, 0 if different)

that is,

$\int_{\mathbb{R}} \psi_a \cdot \bar{\psi}_b \, dx = 0$ if $a \neq b$, but $\int_{\mathbb{R}} \psi_a \cdot \bar{\psi}_a \, dx = 1$

Quantum Operators:

Classically, these are measurable properties of a system.
In quantum mechanics, they are operators applied to the wavefunction $\psi_n(x)$.

position: $\hat{x} = x$

momentum: $\hat{p} = -i\hbar \frac{\partial}{\partial x}$

energy: $$\hat{H} = -\frac{\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} + \frac{m\omega^2}{2} \cdot x^2$$ (the "Hamiltonian" in Schrödinger)

(KE) (PE for the quantum harmonic oscillator)

Note that the KE term is just $\frac{\hat{p}^2}{2m}$. The PE term depends on the situation.

Expected Value:

Classically, if a random variable x is associated with a probability density $f(x)$, then

P(x is between a and b) = $\int_a^b f(x) \, dx$ and expected value = $\int_{\mathbb{R}} x \cdot f(x) \, dx$.

The quantum approach involves treating $(\psi_n, \psi_n) = \psi_n \cdot \bar{\psi}_n$ as a probability density!

Expected value of an operator $\hat{O}$ in state n is written as $<\hat{O}>_n$ and is calculated as

$$<\hat{O}>_n = (\hat{O}\psi_n, \psi_n) = \int_{\mathbb{R}} (\hat{O}\psi_n) \cdot (\bar{\psi}_n) \, dx.$$

Try it for $<\hat{p}>_n$ and $<\hat{x}>_n$ and see what happens!