Math 22A: Linear Algebra Practice Final

1) Let $A = \begin{bmatrix} 2 & 2 & 1 & 0 & 1 \\ 3 & 3 & 4 & 5 & 3 \\ 2 & 2 & 2 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$.

Find bases for A's row space, column space, and null space.

$R(A) = \text{span}\left\{ \qquad\qquad\qquad\qquad \right\}$

$C(A) = \text{span}\left\{ \qquad\qquad\qquad\qquad \right\}$

$N(A) = \text{span}\left\{ \qquad\qquad\qquad\qquad \right\}$

What is the rank of A?

What is the nullity of A?

What do these add up to, and why?

Without actually finding the left null space of A, what is its dimension, and why?

Choose a vector in R(A) and a vector in N(A), and calculate their dot product.
Why *must* the result be that value?

Find the complete solution of $A \cdot \vec{x} = \begin{bmatrix} 6 \\ 4 \\ 11 \\ 3 \end{bmatrix}$.

2) Find a line $y=mx+b$ (that is, find *m* and *b*) of best fit for this data set:

x	y
1	-1
2	0
3	5
4	6

3) Let $\vec{u}=\begin{bmatrix}-3\\4\\-4\\3\end{bmatrix}$ and $W=\text{span}\left\{\begin{bmatrix}0\\0\\1\\2\end{bmatrix},\begin{bmatrix}0\\1\\2\\3\end{bmatrix},\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\right\}$.

Find an orthogonal basis for *W*. (You can normalize it too if you want practice.)

Find the projection of $\vec{u}$ onto *W*.

4) Is the set of all cubics *without* an x^2 term a vector space? Why or why not?

Is the set of all functions that pass through (2,3) a vector space? Why or why not?

5) Find all eigenvalues and eigenvectors of $B=\begin{bmatrix} 1 & 0 & -2 \\ -1 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$. (Bonus: diagonalize B.)

6) Factor the matrix $C=\begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ into LPU or PLU form (your choice).

Why is it not possible to factor it into just LU form?

7) Let $G=\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$. Evaluate det(G).

Is G invertible? If it is, find det(G^{-1}) without finding G^{-1}.

Evaluate det(10 G) with as little work as possible.

8) Let $H = \begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix}$. Find the LDU-decomposition of H.

9) Prove that if $A\vec{x} = \vec{0}$ has only the trivial solution, then $A\vec{x} = \vec{b}$ has no more than one solution.

10) Let T be a transformation matrix that takes vectors in $\mathbf{R}^2$ as input and produces vectors in $\mathbf{R}^3$ as output. Given the following input-output pairs, find matrix T.

$$T \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T \cdot \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 3 \end{bmatrix}$$