Math 22A: Linear Algebra Practice Final

1) Let $A = \begin{bmatrix} 2 & 2 & 1 & 0 & 1 \\ 3 & 3 & 4 & 5 & 3 \\ 2 & 2 & 2 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$.

Find bases for A's row space, column space, and null space.

$R(A) = \text{span}\left\{ \qquad \qquad \qquad \qquad \right\}$

$C(A) = \text{span}\left\{ \qquad \qquad \qquad \qquad \right\}$

$N(A) = \text{span}\left\{ \qquad \qquad \qquad \qquad \right\}$

What is the rank of A?

What is the nullity of A?

What do these add up to, and why?

Without actually finding the left null space of A, what is the left nullity of A, and why?

Choose a vector in R(A) and a vector in N(A), and calculate their dot product.
Why *must* the result be that value?

Find the complete solution of $A \cdot \vec{x} = \begin{bmatrix} 6 \\ 4 \\ 11 \\ 3 \end{bmatrix}$.

2) Find a line $y = mx + b$ (that is, find *m* and *b*) of best fit for this data set:

x	y
1	-1
2	0
3	5
4	6

3) Let $\vec{u} = \begin{bmatrix} -3 \\ 4 \\ -4 \\ 3 \end{bmatrix}$ and $W = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

Find the projection of $\vec{u}$ onto *W*.

Find an orthogonal basis for *W*. (You can normalize it too if you want practice.)

4) Find all eigenvalues and eigenvectors of $\begin{bmatrix} -11 & -2 & 5 \\ 3 & 0 & -3 \\ -8 & -2 & 2 \end{bmatrix}$.

5) Find the solutions to this system of differential equations:

$$\frac{dx}{dt} = -11x - 2y + 5z$$

$$\frac{dy}{dt} = 3x - 3z$$

$$\frac{dz}{dt} = -8x - 2y + 2z$$

(Hint: you've already done most of the work for this.)

6) Let T be a linear transform from $\mathbb{R}^3$ to $\mathbb{R}^4$ such that

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}2\\3\\0\\1\end{bmatrix},\quad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}-1\\0\\-5\\6\end{bmatrix},\quad \text{and}\quad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}0\\1\\1\\1\end{bmatrix}.$$

Find $T\left(\begin{bmatrix}2\\-3\\4\end{bmatrix}\right)$.

Find a matrix that is equivalent to T; that is,
a matrix M such that for any $\vec{u}\in\mathbb{R}^3$, $\mathrm{M}\cdot\vec{u}=T(\vec{u})$.

7) In differential equations, you will learn about the Laplace transform, which takes a function of t as input and produces a different function of s as output. It is defined as:

$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)\cdot e^{-st}\,\mathrm{d}t$, where s is essentially a placeholder variable.

Is $\mathcal{L}$ a linear transform? Why or why not?

8) Find the singular value decomposition (SVD) of the matrix $\mathrm{A}=\begin{bmatrix}1&1&0\\0&0&1\end{bmatrix}$;

that is, rewrite A as $\mathrm{U}\cdot\Sigma\cdot\mathrm{V}^{\mathrm{T}}$. Note that V^{T} should equal V^{-1}.